TECHNISCHE UNIVERSITÄT KAISERSLAUTERN
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Computer Algebra

Summer Term 2019 - Sheet 9

Due Date: Wednesday, June 19, 2019, 10:00 am

Exercise 29. Show by example that Algorithm 2.5.16 in the book by Greuel and Pfister does not return a minimal free resolution in general.

Exercise 30. Let A be a local ring and M a finitely generated A-module with presentation

$$A^m \xrightarrow{\varphi} A^n \longrightarrow M \longrightarrow 0,$$

where $\varphi \in \mathrm{Mat}(n \times m, A)$. Furthermore, let $\psi : A^m \to A^m$ and $\chi : A^n \to A^n$ be isomorphisms.

(a) Prove that $\mathrm{Ker}(\varphi) \cong \mathrm{Ker}(\chi \circ \varphi \circ \psi)$ and $\mathrm{Im}(\varphi) \cong \mathrm{Im}(\chi \circ \varphi \circ \psi)$.

(b) Assume there exists a fixed pair (i, j), such that $\varphi_{i,j} = 1, \varphi_{i',j} = 0$ for all $i' \neq i$ and $\varphi_{i,j'} = 0$ for all $j' \neq j$. Denote by $\tilde{\varphi}$ the $(n-1) \times (m-1)$ matrix resulting from removing the i-th row and j-th column of φ. Show that $\mathrm{Coker}(\varphi) \cong \mathrm{Coker}(\tilde{\varphi})$.

(c) State an algorithm to compute a minimal presentation of M from any given presentation of M and prove the correctness of your algorithm.

Hint: See p. 127 in the book by Greuel and Pfister.

Exercise 31. Let $>$ be a local ordering, $A = K[x]_>$ and $I \subseteq A^r$ be a finitely generated A-module. Furthermore, let the matrices $A_1, \dots, A_m$ be the output of Algorithm 2.5.16 in the book by Greuel and Pfister applied to I.
State an algorithm that computes a minimal free resolution of A^r/I and prove its correctness.
Hint: See p. 166 in the book by Greuel and Pfister.

Exercise 32. Write a SINGULAR procedure `stdBuchberger(ideal I)` to compute the Standard basis of a given ideal $I \trianglelefteq K[x_1, \dots, x_n]_>$ with respect to any monomial ordering $>$ without the use of the commands `std` and `groebner`.