TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

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Fachbereich Mathematik

Computer Algebra

Summer Term 2019 - Sheet 8

Due Date: Thursday, June 13, 2019, 10:00 am

Exercise 26. Consider the local ring $A = \mathbb{Q}[x, y, z, w]_{\langle x,y,z,w\rangle}$ and the A-modules

$$M_1 = A/\langle yz - xw, z^3 - yw^2, xz^2 - y^2w, y^3 - x^2z\rangle \text{ and } M_2 = A/\langle xyz, yzw, zwx, wxy\rangle.$$

Can M_1 and M_2 be isomorphic as A-modules? Prove your claim.

Exercise 27. Let A be a local ring and M a finitely generated A-module with minimal generating sets $\{f_1, \ldots, f_k\}$ and $\{g_1, \ldots, g_k\}$. Prove that $\operatorname{syz}(f_1, \ldots, f_k) \cong \operatorname{syz}(g_1, \ldots, g_k)$.

Exercise 28. Let R a commutative ring with unit and let A_i, B_i for $i = 1, \ldots, 5$ be R-modules. Consider the following commutative diagram:

$$\begin{array}{ccccccccc} A_1 & \xrightarrow{\varphi_1} & A_2 & \xrightarrow{\varphi_2} & A_3 & \xrightarrow{\varphi_3} & A_4 & \xrightarrow{\varphi_4} & A_5 \\ \downarrow f_1 & & \cong \downarrow f_2 & & \downarrow f_3 & & \cong \downarrow f_4 & & \downarrow f_5 \\ B_1 & \xrightarrow{\psi_1} & B_2 & \xrightarrow{\psi_2} & B_3 & \xrightarrow{\psi_3} & B_4 & \xrightarrow{\psi_4} & B_5 \end{array}$$

Assume that the rows are exact, f_1 is an epimorphism, f_2 and f_4 are isomorphisms and that f_5 is a monomorphism. Show that f_3 is an isomorphism.