TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

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Computer Algebra

Summer Term 2019 - Sheet 4

Due Date: Thursday, May 16, 2019, 10:00 am

Exercise 12.

(a) Let $>$ be any monomial ordering, $R = K[x_1, \ldots, x_n]_>$ and $I \trianglelefteq R$ an ideal. Show that if I has a reduced standard basis, then it is unique.

(b) Show that Remark 1.7.2 in the SINGULAR book is not correct.

Exercise 13.

(a) Show by example that reduced normal forms with respect to non-global orderings do not exist in general.

(b) Let $>$ be the ordering `ds`. Compute a standard representation of x_1 with respect to $\{x_1 - x_2, x_2 - x_1^2\}$ in $K[x_1, x_2]_>$.

Exercise 14 (Product Criterion)**.**
Let $>$ be a global monomial ordering on $\mathrm{Mon}(x_1, \ldots, x_n)$. Let $f, g \in K[x_1, \ldots, x_n]$ be polynomials such that $\mathrm{lcm}(\mathrm{LM}_>(f), \mathrm{LM}_>(g)) = \mathrm{LM}_>(f) \cdot \mathrm{LM}_>(g)$. Prove that

$$\mathrm{NF}\left(\mathrm{spoly}(f, g) \mid \{f, g\}\right) = 0.$$

Hint: Assume that $\mathrm{LC}_>(f) = \mathrm{LC}_>(g) = 1$ and claim that $\mathrm{spoly}(f, g) = -\,\mathrm{tail}(g) \cdot f + \mathrm{tail}(f) \cdot g$ is a standard representation.