TECHNISCHE UNIVERSITÄT
KAISERSLAUTERN

Prof. Dr. Mathias Schulze
Raul Epure
Fachbereich Mathematik

Computer Algebra

Summer Term 2019 - Sheet 3

Due Date: Thursday, May 09, 2019, 10:00 am

Exercise 8. Let $>$ be an ordering on $\mathrm{Mon}(x_1,\dots,x_n)$ and $\mathfrak{p} \trianglelefteq K[x_1,\dots,x_n]$ a prime ideal such that $K[x_1,\dots,x_n]_> = K[x_1,\dots,x_n]_{\mathfrak{p}}$. Prove that $\mathfrak{p}$ is a monomial ideal, i.e. that it can be generated by monomials.

Exercise 9. For a polynomial $f = \sum_{\alpha\in\mathbb{N}^n} c_\alpha \cdot x_1^{\alpha_1}\dots x_n^{\alpha_n} \in K[x_1,\dots,x_n]$ and for an ideal $I \trianglelefteq K[x_1,\dots,x_n]$ we define their homogenizations as

$$\begin{aligned} f^h &:= \sum_{\alpha\in\mathbb{N}^n} c_\alpha \cdot x_0^{\deg(f)-|\alpha|} x_1^{\alpha_1}\dots x_n^{\alpha_n} \in K[x_0,\dots,x_n], \\ I^h &:= \langle f^h \mid f \in I\rangle \trianglelefteq K[x_0,\dots,x_n]. \end{aligned}$$

For an ordering $>$ on $\mathrm{Mon}(x_1,\dots,x_n)$ defined by a matrix $A \in \mathrm{GL}(n,\mathbb{Q})$ let $>_h$ be the ordering on $\mathrm{Mon}(x_0,\dots,x_n)$ defined by the matrix

$$\begin{pmatrix} 1 & 1 & \cdots & 1 \\ 0 & & & \\ \vdots & & A & \\ 0 & & & \end{pmatrix}.$$

Now let $\{G_1,\dots,G_k\}$ be a homogeneous (i.e. each G_i only has terms of a fixed degree) standard basis of $I^h \trianglelefteq K[x_0,\dots,x_n]$ with respect to $>_h$. Prove that $\{G_1|_{x_0=1},\dots,G_k|_{x_0=1}\}$ is a standard basis for I with respect to $>$.

Exercise 10. Let $>$ be a global degree ordering on $\mathrm{Mon}(x_1,\dots,x_n)$, and let $\{g_1,\dots,g_k\}$ be a Gröbner basis of the ideal $I \trianglelefteq K[x_1,\dots,x_n]$ with respect to $>$. Prove that $I^h = \langle g_1^h,\dots,g_k^h\rangle$.

Exercise 11. Write a Singular procedure `powSeriesInv(poly f, int n)` that, having as input a polynomial $f \in K[x]$ over a field K, and an integer $n \in \mathbb{N}$, returns the power series expansion of the inverse of f up to terms of degree n if f is a unit in $K[x]_>$ (where $>$ is the monomial ordering of the basering $K[x]$) and 0 if f is not a unit.