TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

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Commutative Algebra

Winter Semester 2016 - Problem Set 11

Due January 27, 2017, 1 p.m.

Problem 1:

(a) Reduce Proposition 6.23 to the case where R is reduced.

(b) Let $\varphi : R \to S$ be flat and M an R-module. Show that $\varphi(M^{\mathrm{reg}}) \subseteq (M_S)^{\mathrm{reg}}$.

Problem 2: Let R, S and T be rings. Show:

(a) If $R \to S$ is finite, then it is of finite type. The converse statement is not true.

(b) If $R \to S$ and $S \to T$ are finite, then $R \to T$ is finite. An analogous statement holds for finite type.

Problem 3: Let $R \leq S$ be integral domains and $f, g \in S[X]$ be monic polynomials. Show that if $f \cdot g \in \overline{R}^S[X]$, then $f, g \in \overline{R}^S[X]$.

Problem 4: Show that the going down property does in general not hold for integral extensions.
Hint: Consider the ring extension

$$R := \{f \in S \mid f(0,0) = f(1,1)\} \leq S := k[X,Y]$$

(where k is a field) and the ideals $\mathfrak{q}' := \langle X-1, Y-1\rangle \in \operatorname{Spec}(S)$ and $\mathfrak{p} := \langle X\rangle \cap R$, $\mathfrak{p}' := \mathfrak{q}' \cap R \in \operatorname{Spec}(R)$. Show that there is no $\mathfrak{q} \in \operatorname{Spec}(S)$ with $\mathfrak{q} \leq \mathfrak{q}'$ lying over $\mathfrak{p}$.