Technische Universität Kaiserslautern

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Computer Algebra

Winter Semester 2015 - Problem Set 5

Due December 3, 2015, 10:00

Problem 1: Check by hand whether the following inclusions are correct:

(a) $xy^3 - z^2 + y^5 - z^3 \in \langle -x^3 + y, x^2y - z\rangle \trianglelefteq \mathbb{Q}[x, y, z]$

(b) $x^3z - 2y^2 \in \langle yz - y, xy + 2z^2, y - z\rangle \trianglelefteq \mathbb{Q}[x, y, z]$

(c) $x^3z - 2y^2 \in \langle yz - y, xy + 2z^2, y - z\rangle \trianglelefteq \mathbb{Q}[x, y, z]_{\langle x,y,z\rangle}$

Problem 2: Let $>$ be a global monomial ordering on $\mathrm{Mon}(x_1, \ldots, x_n)$, let $I \trianglelefteq K[x_1, \ldots, x_n]$ be an ideal, and let G be a standard basis of I with repsect to $>$. Show that the following are equivalent:

(a) $\dim_K K[x_1, \ldots, x_n]/I < \infty$,

(b) for all $i = 1, \ldots, n$ there exists an $l \in \mathbb{N}$ such that $x_i^l = \mathrm{LM}_>(g)$ for a $g \in G$.

Problem 3:

(a) Let $0 \neq I \subseteq K[x_1, \ldots, x_n]$ be an ideal, and let $>$ denote the negative lexicographical ordering `ls`.

  (i) Does the highest corner $\mathrm{HC}(I)$ always exist?

  (ii) Assume that x^α, $\alpha = (\alpha_1, \ldots, \alpha_n)$ is the highest corner of I. Show that, for $i = 1, \ldots, n$,
$$\alpha_i = \max\{p \mid x_1^{\alpha_1} \cdots x_{i-1}^{\alpha_{i-1}} x_i^p \notin L(I)\}.$$

(b) Compute the highest corner of $I = \langle x^2 + x^2y, y^3 + xy^3, z^3 - xz^2\rangle$ with respect to the orderings `ls` and `ds` by hand.

Problem 4: Implement an own Gröbner basis algorithm in Singular.