Technische Universität Kaiserslautern

Prof. Dr. Mathias Schulze
Dipl.-Math. Cornelia Rottner
Fachbereich Mathematik

Computer Algebra

Winter Semester 2015 - Problem Set 3

Due November 19, 2015, 10:00

Problem 1: Let $>$ be an ordering on $\mathrm{Mon}(x_1, \dots, x_n)$ and $\mathfrak{p} \trianglelefteq K[x_1, \dots, x_n]$ a prime ideal such that $K[x_1, \dots, x_n]_> = K[x_1, \dots, x_n]_{\mathfrak{p}}$. Prove that $\mathfrak{p}$ is a monomial ideal, i.e. that it can be generated by monomials.

Problem 2: For a polynomial $f = \sum_{\alpha \in \mathbb{N}^n} c_\alpha \cdot x_1^{\alpha_1} \dots x_n^{\alpha_n} \in K[x_1, \dots, x_n]$ and for an ideal $\mathfrak{a} \trianglelefteq K[x_1, \dots, x_n]$ we define their homogenizations as

$$f^h := \sum_{\alpha \in \mathbb{N}^n} c_\alpha \cdot x_0^{\deg(f) - |\alpha|} x_1^{\alpha_1} \dots x_n^{\alpha_n} \in K[x_0, \dots, x_n],$$
$$\mathfrak{a}^h := \langle f^h \mid f \in \mathfrak{a} \rangle \trianglelefteq K[x_0, \dots, x_n].$$

For an ordering $>$ on $\mathrm{Mon}(x_1, \dots, x_n)$ defined by a matrix $A \in \mathrm{GL}(n, \mathbb{Q})$ let $>_h$ be the ordering on $\mathrm{Mon}(x_0, \dots, x_n)$ defined by the matrix

$$\begin{pmatrix} 1 & 1 & \cdots & 1 \\ 0 & & & \\ \vdots & & A & \\ 0 & & & \end{pmatrix}.$$

Now let $\{G_1, \dots, G_k\}$ be a homogeneous (i.e. each G_i only has terms of a fixed degree) standard basis of $\mathfrak{a}^h \trianglelefteq K[x_0, \dots, x_n]$ with respect to $>_h$. Prove that $\{G_1|_{x_0=1}, \dots, G_k|_{x_0=1}\}$ is a standard basis for $\mathfrak{a}$ with respect to $>$.

Problem 3 Let $>$ be a global degree ordering on $\mathrm{Mon}(x_1, \dots, x_n)$, and let $\{g_1, \dots, g_k\}$ be a Gröbner basis of the ideal $\mathfrak{a} \trianglelefteq K[x_1, \dots, x_n]$ with respect to $>$. Prove that $\mathfrak{a}^h = \langle g_1^h, \dots, g_k^h \rangle$.

Problem 4: Write a Singular procedure `powSeriesInv(poly f, int n)` that, having as input a polynomial $f \in K[x]$ over a field K, and an integer $n \in \mathbb{N}$, returns the power series expansion of the inverse of f up to terms of degree n if f is a unit in $K[x]_>$ (where $>$ is the monomial ordering of the basering $K[x]$) and 0 if f is not a unit.