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Computer Algebra

Winter Semester 2015 - Problem Set 2

Due November 12, 2015, 10:00 am

Problem 1 (4 Points). Let $R = K[[x_1, \ldots, x_n]]$ be a power series ring over a field K and $<$ a monomial ordering on $\mathrm{Mon}_n := \{x^\alpha \mid \alpha \in \mathbb{N}^n\}$. Similarly to the case of polynomial rings, one would like to define the leading monomial $\mathrm{LM}(f)$ of any power series $0 \neq f = \sum_\alpha a_\alpha x^\alpha \in R$ by

$$\mathrm{LM}(f) := \max\{x^\alpha \mid a_\alpha \neq 0\}.$$

When is this definition well-defined?

Problem 2 (4 Points). Let $n > 1$ and let $w_1, \ldots, w_n \in \mathbb{R}$ be linearly independent over $\mathbb{Q}$. Define $>$ on Mon_n by setting $x^\alpha < x^\beta$ if $\sum_{i=1}^n w_i\alpha_i < \sum_{i=1}^n w_i\beta_i$.

(a) Prove that $>$ is a monomial ordering.

(b) Show that there is no matrix $A \in \mathrm{GL}(n, \mathbb{Q})$ defining this ordering.

Problem 3 (4 Points).

(a) Consider a matrix ordering $>_A$ on Mon_n for some matrix $A \in \mathrm{GL}(n, \mathbb{Q})$. Let $M \subset \mathrm{Mon}_n$ be a finite set. Determine a weight vector $w \in \mathbb{Z}^n$ which induces $>_A$ on M.
Hint: Use the fact that $x^\alpha >_A x^\beta$ if and only if $A\alpha >_{lex} A\beta$ and Example 1.2.12.

(b) Determine integer weight vectors which induce `dp`, respectively `ds`, on $M := \{x_1^i x_2^j x_3^k \mid 1 \leq i, j, k \leq 5\} \subset \mathrm{Mon}_3$.

Problem 4 (4 Points). Write a Singular procedure `checkWeightVector(list M, intvec v)`, having a list `M` of monomials and an intvec `v` as input and returning `1` if `v` induces the monomial ordering of the basering on the set of monomials contained in `M` and `0` else.

Your procedure is supposed to work as follows: It creates a polynomial f, being the sum of all monomials contained in `M`, and converts f to a string. Then it does the same in other suitable rings (hint: extra weight vectors), and compares the respective strings.

Prove that your procedure is correct.