TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

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Algebraic Geometry

Summer Semester 2015 - Problem Set 9

Due June 19, 2015, 11:00 am

In all exercises, the ground field k is assumed to be algebraically closed.

Problem 1. Let $k = \mathbb{C}$. Sketch the set of real points of the complex affine curve $X = V(x_1^3 - x_1x_2^2 + 1) \subset \mathbb{A}^2$ and compute the points at infinity of its projective closure $\overline{X} \subset \mathbb{P}^2$.

Problem 2.

(a) Read carefully through items 7.27 to 7.30 of Andreas Gathmann's lecture notes.

(b) We let $F : \mathbb{P}^n \to \mathbb{P}^N$ be the d-th Veronese embedding. Find explicit equations describing $X = F(\mathbb{P}^n) = V(I)$, i.e. generators for a homogeneous ideal I with $V(I) = X$.

Problem 3. Let $f : \mathbb{P}^n \to \mathbb{P}^m$ be a morphism. Prove:

(a) If $X \subset \mathbb{P}^m$ is the zero locus of a single homogeneous polynomial in $k[x_0, ..., x_m]$ then every irreducible compoment of $f^{-1}(X)$ has dimension at least $n - 1$.

(b) If $n > m$ then f must be constant.

Problem 4. Let $m, n > 0$ positive numbers. Use Problem 3(b) from Problem Set 8 to show that $\mathbb{P}^n \times \mathbb{P}^m$ is not isomorphic to $\mathbb{P}^{n+m}$.