Technische Universität Kaiserslautern

Prof. Dr. Mathias Schulze
B.Sc. Lucas Ruhstorfer
Fachbereich Mathematik

Algebraic Geometry

Summer Semester 2015 - Problem Set 8

Due June 12, 2015, 11:00 am

In all exercises, the ground field k is assumed to be algebraically closed.

Problem 1.

(a) Prove that a graded ring R is an integral domain if and only if for all homogenenous $f, g \in R$ with $fg = 0$ we have $f = 0$ or $g = 0$.

(b) Show that a projective variety X is irreducible if and only if its homogeneous coordinate ring $S(X)$ is an integral domain.

Problem 2. If $X, Y \subset \mathbb{A}^n$ are two pure-dimensional affine varieties then every irreducible component of $X \cap Y$ has dimension at least $\dim(X)+\dim(Y) - n$.

Hint : Use Problem 3 (a) from Problem Set 4.

Problem 3. Let $\emptyset \neq X, Y \subset \mathbb{P}^n$ projective varieties.

(a) Show that the cone $C(X) \subset \mathbb{A}^{n+1}$ has dimension $\dim(X) + 1$.

(b) If $\dim X+\dim Y \geq n$ then $X \cap Y \neq \emptyset$.

Problem 4. Let $L_1, L_2 \subset \mathbb{P}^3$ be two disjoint lines (i.e. 1-dimensional subspaces in the sense of Example 6.12 (b)), and let $a \in \mathbb{P}^3 \setminus (L_1 \cup L_2)$. Show that there is a unique line $L \subset \mathbb{P}^3$ through a that intersects L_1 and L_2.