Technische Universität Kaiserslautern

Prof. Dr. Mathias Schulze
B.Sc. Lucas Ruhstorfer
Fachbereich Mathematik

Algebraic Geometry

Summer Semester 2015 - Problem Set 7

Due June 5, 2015, 11:00 am

In all exercises, the ground field k is assumed to be algebraically closed.

Problem 1. Let X and Y be prevarieties. Show:

(a) If X and Y are varieties, then $X \times Y$ is a variety.

(b) If X and Y are irreducible, then $X \times Y$ is irreducible.

Problem 2. If X and Y are affine varieties then there is a one-to-one correspondence between the set of morphisms $X \to Y$ and the set of k-algebra homomorphisms $\mathscr{O}_Y(Y) \to \mathscr{O}_X(X)$, given by the map $f \to f^*$. Does this statement still hold if

(a) X is a prevariety and Y is an affine variety,

(b) X is an affine variety and Y is a prevariety?

Problem 3. Show:

(a) If X is a variety, then $\Delta(X) \cong X$.

(b) The intersection of any two affine open subprevarieties of a variety is again an affine open subprevariety.

Hint: Explicitly distinguish different prevariety structures and use results from the lecture (notes) to argue that they agree.

Problem 4. Let $\mathscr{F}$ be a presheaf on a topological space X. For each open set $U \subset X$ we let $\mathscr{F}_{disc}(U) = \{s : U \to \bigcup_{p \in U} \mathscr{F}_p \mid s(p) \in \mathscr{F}_p \text{ for all } p \in U\}$. (This should remind you of Problem 4 on Problem Set 5).

We now define $\mathscr{F}_{sh}(U)$ to be the elements $s \in \mathscr{F}_{disc}(U)$ such that for each $p \in U$ there is an open neighborhood $p \in V \subset U$ and an element $t \in \mathscr{F}(V)$, such that for all $q \in V$, the germ t_q equals $s(q)$. We say that $\mathscr{F}_{sh}$ is the *sheafification* of the presheaf $\mathscr{F}$.

(a) Show that $\mathscr{F}_{sh}$ is a sheaf on X and compute its stalks.

(b) Describe the sheafification of the presheaf of constant functions.