TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

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Algebraic Geometry

Summer Semester 2015 - Problem Set 2

Due April 30, 2015, 11:00 am

In all exercises, the ground field k is assumed to be algebraically closed.

Problem 1.

(a) Let $Y = V(y - x^2) \subset \mathbb{A}^2$. Show that $A(Y)$ is isomorphic to a polynomial ring in one variable.

(b) Let $Z = V(xy - 1)$. Show that $A(Z)$ is not isomorphic to $A(Y)$.

(c) Let $W = V(f)$ for some irreducible quadratic polynomial in $k[x, y]$. Prove that $A(W) \cong A(Z)$ or $A(W) \cong A(Y)$.

Problem 2. Let $X, Y \subset \mathbb{A}^n$ disjoint affine varieties. Prove that $A(X \cup Y) \cong A(X) \times A(Y)$ as k-algebras.

Hint: Chinese Remainder Theorem.

Problem 3. Let X be an affine variety. Show that the coordinate ring $A(X)$ is a field if and only if X is a single point. How does the coordinate ring of an affine variety consiting of only finitely many points look like?

Problem 4. Recall Construction 1.20. Let $Y \subset \mathbb{A}^n$ be an affine variety and let $\pi : k[x_1, \dots, x_n] \to A(Y)$ be the quotient map.

(a) Show that $V_{\mathbb{A}^n}(\pi^{-1}(S)) = V_Y(S)$ for a subset $S \subset A(Y)$.

(b) Show that $I_{\mathbb{A}^n}(X) = \pi^{-1}(I_Y(X))$ for a subset $X \subset Y$.

(c) Use this to prove Proposition 1.21 from the script.