Technische Universität Kaiserslautern

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Algebraic Geometry

Summer Semester 2015 - Problem Set 13

Due July 17, 2015, 11:00 am

In all exercises, the ground field k is assumed to be algebraically closed.

Problem 1. Compute the Hilbert function of

(a) two intersecting lines in $\mathbb{P}^3$

(b) two non-intersecting lines in $\mathbb{P}^3$.

(c) (optional) Compute all possible Hilbert functions for four points in $\mathbb{P}^2$.

Problem 2. Show that the degree of the Segre embedding of $\mathbb{P}^m \times \mathbb{P}^n$ is $\binom{n+m}{n}$.

Problem 3. Prove that every pure-dimensional projective variety of degree 1 is a linear space.

Problem 4. Let $X, Y \subset \mathbb{A}^2$ be two affine curves containing the origin. Let $I(X) = (f)$ and $I(Y) = (g)$. Show that the following statements are equivalent:

(a) $\mathscr{O}_{\mathbb{A}^2,0}/(f,g)$ has k-dimension 1 (i.e. the intersection multiplicity of X and Y is 1 at the origin).

(b) X and Y are smooth at 0 and have different tangent spaces at 0 (i.e. X and Y intersect transversely at the origin).