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# Algebraic Geometry

Summer Semester 2015 - Problem Set 1

Exercises to be discussed in the tutorial.

In all exercises, the ground field $k$ is assumed to be algebraically closed.

**Problem 1.** Determine the radical of the ideal $(x_1^3 - x_2^6, x_1x_2 - x_2^3) \lhd \mathbb{C}[x_1, x_2]$.

**Problem 2.** Show that every affine variety $X \subset \mathbb{A}^n$ consisting of finitely many points is the zero locus of $n$ polynomials.

**Hint:** Interpolation.

**Problem 3.** Prove that every affine variety in the **real** affine space $\mathbb{A}^n_{\mathbb{R}}$ is the zero locus of one polynomial.

**Problem 4.** Let $X = \{(t, t^2, t^3) \mid t \in k\}$. Show that $X$ is an affine variety. What is $A(X)$ isomorphic to?