TECHNISCHE UNIVERSITÄT
KAISERSLAUTERN

Prof. Dr. Mathias Schulze
Dipl.-Math. Cornelia Rottner
Fachbereich Mathematik

Computer Algebra

Winter Semester 2014 - Problem Set 8

Due January 8, 2015, 10:00

Problem 1: Compute in SINGULAR the normalization of $\mathbb{Q}[x,y,z]/\langle z(y^3-x^5)+x^{10}\rangle$ without using the command `normal`.

Problem 2: Let A be a ring. Prove that $\left(\bigcup_{\mathfrak{p}\in\operatorname{Ass}(A)}\mathfrak{p}\right)\setminus\{0\}$ is the set of zerodivisors of A. Moreover, if A is reduced, show that then $\left(\bigcup_{\mathfrak{p}\in\operatorname{Ass}(A)\text{ minimal}}\mathfrak{p}\right)\setminus\{0\}$ is already the set of zero-divisors of A.

Problem 3: Let A be a reduced Noetherian ring, $J\trianglelefteq A$ a test ideal for normality of A, and consider the integral extention $A\hookrightarrow A' := \operatorname{Hom}_A(J,J)$. Show that $J' := \sqrt{J\cdot A'}$ is a test ideal for normality of A'.

HINT: The first two conditions for test ideals are straight forward. For the third condition, show that $\mathfrak{q}\not\supseteq J'$ implies $\mathfrak{p} := \mathfrak{q}\cap A\not\supseteq J$ for any $\mathfrak{q}\trianglelefteq A'$ prime. Then argue that $A_\mathfrak{p}$ normal implies $A'_\mathfrak{q}$ normal.

Problem 4: Let $\psi: R\to S$ be a finite ring map. Find $k\in\mathbb{N}_{\geq 0}$ and $I\trianglelefteq R[t_1,\dots,t_k]$ such that

$$S\cong R[t_1,\dots,t_k]/I.$$