Technische Universität Kaiserslautern

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Computer Algebra

Winter Semester 2014 - Problem Set 4

Due November 27, 2014, 10:00

Problem 1: Check by hand whether the following inclusions are correct:

(a) $xy^3 - z^2 + y^5 - z^3 \in \langle -x^3 + y, x^2y - z \rangle \trianglelefteq \mathbb{Q}[x, y, z]$

(b) $x^3z - 2y^2 \in \langle yz - y, xy + 2z^2, y - z \rangle \trianglelefteq \mathbb{Q}[x, y, z]$

(c) $x^3z - 2y^2 \in \langle yz - y, xy + 2z^2, y - z \rangle \trianglelefteq \mathbb{Q}[x, y, z]_{\langle x,y,z \rangle}$

Problem 2: Let $>$ be a global monomial ordering on $\mathrm{Mon}(x_1, \ldots, x_n)$, let $I \trianglelefteq K[x_1, \ldots, x_n]$ be an ideal, and let G be a standard basis of I with repsect to $>$. Show that the following are equivalent:

1. $\dim_K K[x_1, \ldots, x_n]/I < \infty$,
2. for all $i = 1, \ldots, n$ there exists an $l \in \mathbb{N}$ such that $x_i^l = \mathrm{LM}_>(g)$ for a $g \in G$.

Problem 3: Let $R := K[x_1, \ldots, x_n]$ be a polynomial ring, $>$ a global monomial ordering on $\mathrm{Mon}(x_1, \ldots, x_n)$, and $I \trianglelefteq R$ an ideal generated by $G := \{g_1, \ldots, g_k\}$. Let $F_\bullet^>$ denote the filtration on R induced by the monomial ordering $>$, i.e. $F_\alpha^> R := \bigoplus_{\beta \leq \alpha} K x^\beta$ $(\alpha \in \mathbb{N}^k)$. This filtration and G induce a filtration $F_\bullet^{(>,G)}$ on R^k given by $F_\bullet^{(>,G)} R^k := \bigoplus_{1 \leq i \leq k} F_{\bullet - \alpha_i} R$, where $\alpha_i := \mathrm{LE}(g_i)$. Show that the map

$$F_\bullet^{(>,G)} R^k \xrightarrow{(g_1,\ldots,g_k)} F_\bullet^> R$$

is strict if and only if G is a Gröbner basis of I with respect to $>$.

Problem 4: Implement an own Gröbner basis algorithm in SINGULAR.