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# Computer Algebra

Winter Semester 2014 - Problem Set 3

Due November 20, 2014, 10:00

**Problem 1:** Let $>$ be any monomial ordering, $R = K[x_1, \dots, x_n]_>$, $I \subset R$ an ideal, $G \subset I$ a standard basis of $I$ and $\mathrm{NF}(-, G)$ a normal form on $R$. Show that:

1. If $\mathrm{NF}(-, G)$ is reduced, then it is unique.
2. If $G$ is a reduced, then it is unique.

**Problem 2:**

1. Show by example that reduced normal forms with respect to non-global orderings do in general not exist.
2. Let $>$ be the ordering `ds`. Compute a standard representation of $x_1$ with respect to $\{x_1 - x_2, x_2 - x_1^2\}$ in $K[x_1, x_2]_>$.

**Problem 3:** (*Product Criterion*) Let $>$ be a global monomial ordering on $\mathrm{Mon}(x_1, \dots, x_n)$. Let $f, g \in K[x_1, \dots, x_n]$ be polynomials such that $\mathrm{lcm}(\mathrm{LM}_>(f), \mathrm{LM}_>(g)) = \mathrm{LM}_>(f) \cdot \mathrm{LM}_>(g)$. Prove that

$$\mathrm{NF}(\mathrm{spoly}(f, g) \mid \{f, g\}) = 0.$$

Hint: Assume that $\mathrm{LC}_>(f) = \mathrm{LC}_>(g) = 1$ and claim that $\mathrm{spoly}(f, g) = -\mathrm{tail}(g) \cdot f + \mathrm{tail}(f) \cdot g$ is a standard representation.

**Problem 4:** Write a Singular procedure to compute the reduced normal form of a given polynomial $f \in K[x_1, \dots, x_n]$ with respect to a given finite list of polynomials $G \subseteq K[x_1, \dots, x_n]$ and a global monomial ordering $>$ without the use of the commands `reduce` and `NF`.