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Computer Algebra

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Problem 1: Let A be a Noetherian ring, $Q \trianglelefteq A$ an ideal and M a finitely generated A-module. Given a tuple of generators $(m_1, \ldots, m_k) \in M^k$ and a tuple of shifts $(n_1, \ldots, n_k) \in \mathbb{Z}^k$, one can define a Q-filtration on M by

$$M_n := \sum_{i=1}^{k} Q^{n+n_i} \cdot m_i \text{ for all } n \in \mathbb{Z}, \text{ where } Q^l := 0 \text{ for } l < 0,$$

effectively making the following representation strict:

$$\begin{array}{ccccc} A^k & \longrightarrow & M & \longrightarrow & 0 \\ \cup\!\mid & e_i \mapsto m_i & \cup\!\mid & & \\ (A^k)_n := \bigoplus_{i=1}^{k} Q^{n+n_i} & \longrightarrow & M_n = \sum_{i=1}^{k} Q^{n+n_i} \cdot m_i & & \end{array}$$

Show that any Q-stable filtration on M is of this form.

Problem 2: Let A be a Noetherian local ring, $I \trianglelefteq A$ an ideal and $I = Q_1 \cap \ldots \cap Q_m$ its irredundant primary decomposition with $\dim(A/I) = \dim(A/Q_i)$ for $i = 1, \ldots, s$ and $\dim(A/I) > \dim(A/Q_j)$ for $j = s+1, \ldots, r$. Prove that $\text{mult}(A/I) = \sum_{i=1}^{s} \text{mult}(A/Q_i)$.

HINT: compare with Lemma 5.3.11 in [GP][1].

Problem 3: Write a SINGULAR prodecure to compute the Hilbert-Samuel polynomial of $K[x]_{\langle x \rangle}/I \cdot K[x]_{\langle x \rangle}$, where $x = (x_1, \ldots, x_n)$, for a given homogeneous ideal $I \trianglelefteq K[x]$ using Proposition 5.5.5 and 5.5.7 in [GP].

[1] Gert-Martin Greuel, Gerhard Pfister: "A SINGULAR Introcution to Commutative Algebra"