TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

Prof. Dr. Mathias Schulze
Dipl.-Math. Yue Ren
Fachbereich Mathematik

Computer Algebra

Winter Semester 2013 - Problem Set 9

Due January 14, 2014, 12:00

Problem 1: Let $\mathfrak{a} \trianglelefteq \mathbb{Q}[x]$ be a homogeneous ideal and $\mathfrak{a}_0 := \mathfrak{a} \cap \mathbb{Z}[x]$, where $x = (x_1, \ldots, x_n)$. Let $p \in \mathbb{N}$ be a prime number, $\mathfrak{a}_p := \overline{\mathfrak{a}}_0 \trianglelefteq \mathbb{F}_p[x]$. Prove that

$$H_{\mathbb{Q}[x]/\mathfrak{a}}(n) \leq H_{\mathbb{F}_p[x]/\mathfrak{a}_p}(n) \text{ for all } n \in \mathbb{N}$$

Problem 2: Let $H : \mathbb{N} \to \mathbb{Z}$ be any function. Show that the following are equivalent:

(i) $H(n)$ is a polynomial of degree d for sufficiently large n,

(ii) $D(n) := H(n+1) - H(n)$ is a polynomial of degree $d-1$ for sufficiently large n,

(iii) $S(n) := \sum_{i=0}^{n-1} H(i)$ is a polynomial of degree $d+1$ for sufficiently large n.

Moreover, if $\frac{a_d}{d!}$ is the leading coefficient of H, then D and S have leading coefficient $\frac{a_d}{(d-1)!}$ and $\frac{a_d}{(d+1)!}$ respectively.

Problem 3: Write a SINGULAR prodecure to compute the Hilbert polynomial of $K[x_1, \ldots, , x_n]/I$ for a given homogeneous ideal I.