Technische Universität Kaiserslautern

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Computer Algebra

Winter Semester 2013 - Problem Set 7

Due December 17, 2013, 12:00

Problem 1: Compute in Singular the normalization of $\mathbb{Q}[x,y,z]/\langle z(y^3-x^5)+x^{10}\rangle$ without using the command `normal`.

Problem 2: Let A be a ring. Prove that $\bigcup_{\mathfrak{p}\in\mathrm{Ass}(\langle 0\rangle)}\mathfrak{p}$ is the set of zerodivisors of A. Moreover, if A is reduced, then $\bigcup_{\mathfrak{p}\in\mathrm{Ass}(\langle 0\rangle)\text{ minimal}}\mathfrak{p}$ is already the set of zerodivisors of A.

Problem 3: Let A be a reduced Noetherian ring, $J \trianglelefteq A$ a test ideal for normality of A, and consider the integral extention $A \hookrightarrow A' := \mathrm{Hom}_A(J,J)$. Show that $J' := \sqrt{J\cdot A'}$ is a test ideal for normality of A'.

Hint: The first two conditions for test ideals are straight forward. For the third condition, show that $J' \cap A = J$ and use that to prove that $\mathfrak{q} \not\subseteq J'$ implies $\mathfrak{p} := \mathfrak{q}\cap A \not\subseteq J$ for any $\mathfrak{q} \trianglelefteq A'$ prime. Then argue how $A_{\mathfrak{p}}$ normal implies $A'_{\mathfrak{q}}$ normal.

Problem 4: Write a Singular prcedure to compute the ecard of a given polynomial and use this to implement a normal form algorithm for non-global orderings.