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Computer Algebra

Winter Semester 2013 - Problem Set 4

Due November 26, 2013, 12:00

Problem 1: (*Computing ideal quotients*) Let $>$ be a monomial ordering on $\mathrm{Mon}(x_1, \ldots, x_n)$, let $\mathfrak{a}, \mathfrak{b} \trianglelefteq K[x_1, \ldots, x_n]_>$ be two ideals in the localized polynomial ring with $\mathfrak{a} = \langle g_1, \ldots, g_r \rangle$, $\mathfrak{b} = \langle h_1, \ldots, h_s \rangle$, $g_i, h_j \in K[x_1, \ldots, x_n]$. Define $h := h_1 + t \cdot h_2 + \ldots + t^{r-1} h_r \in K[t, x_1, \ldots, x_n]$. Prove that

$$\mathfrak{a} : \mathfrak{b} = \langle (\langle g_1, \ldots, g_r \rangle_{K[t,x_1,\ldots,x_n]} : h) \cap K[x_1, \ldots, x_n] \rangle_{K[x_1,\ldots,x_n]_>}.$$

Problem 2: Let $>$ be a global monomial ordering on $\mathrm{Mon}(x_1, \ldots, x_n)$, let $I \trianglelefteq K[x_1, \ldots, x_n]$ be an ideal, and let G be a standard basis of I with repsect to $>$. Show that the following are equivalent:

1. $\dim_K K[x_1, \ldots, x_n]/I < \infty$,
2. for all $i = 1, \ldots, n$ there exists an $l \in \mathbb{N}$ such that $x_i^l = \mathrm{LM}_>(g)$ for a $g \in G$.

Problem 3: (*Zariski closure of the image*) Consider $\varphi : \mathbb{Q}^2 \to \mathbb{Q}^4$, $(s, t) \mapsto (s^4, s^3 t, s t^3, t^4)$. Compute the Zariski closure of the image, $\overline{\varphi(\mathbb{Q}^2)}$, and decide whether $\varphi(\mathbb{Q}^2)$ coincides with its closure.

Problem 4: Implement an own Gröbner basis algorithm in SINGULAR.