TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

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Computer Algebra

Winter Semester 2013 - Problem Set 2

Due November 12, 2013, 12:00

Problem 1: Let $>$ be an ordering on $\mathrm{Mon}(x_1,\dots,x_n)$ and $\mathfrak{p} \trianglelefteq K[x_1,\dots,x_n]$ a prime ideal such that $K[x_1,\dots,x_n]_> = K[x_1,\dots,x_n]_\mathfrak{p}$. Prove that $\mathfrak{p}$ is a monomial ideal, i.e. that it can be generated by monomials.

Problem 2: For a polynomial $f = \sum_{\alpha\in\mathbb{N}^n} c_\alpha \cdot x_1^{\alpha_1}\dots x_n^{\alpha_n} \in K[x_1,\dots,x_n]$ and for an ideal $\mathfrak{a} \trianglelefteq K[x_1,\dots,x_n]$ let

$$f^h := \sum_{\alpha\in\mathbb{N}^n} c_\alpha \cdot x_0^{\deg(f)-|\alpha|} x_1^{\alpha_1}\dots x_n^{\alpha_n} \in K[x_0,\dots,x_n],$$
$$\mathfrak{a}^h := \langle f^h \mid f \in I\rangle \trianglelefteq K[x_0,\dots,x_n].$$

Now let $>$ be a global degree ordering, and let $\{g_1,\dots,g_k\}$ be a Gröbner basis of $\mathfrak{a}$. Prove that $\mathfrak{a}^h = \langle g_1^h,\dots,g_k^h\rangle$.

Problem 3: For an ordering $>$ on $\mathrm{Mon}(x_1,\dots,x_n)$ defined by a matrix $A \in \mathrm{GL}(n,\mathbb{Q})$ let $>_h$ be the ordering on $\mathrm{Mon}(x_0,\dots,x_n)$ defined by the matrix

$$\begin{pmatrix} 1 & 1 & \cdots & 1 \\ 0 & & & \\ \vdots & & A & \\ 0 & & & \end{pmatrix}.$$

Let $\mathfrak{a} \trianglelefteq K[x_1,\dots,x_n]$ be an ideal and let $\{G_1,\dots,G_k\}$ a homogeneous (i.e. each G_i only has terms of a fixed degree) standard basis of $\mathfrak{a}^h \trianglelefteq K[x_0,\dots,x_n]$ with respect to $>_h$. Prove that $\{G_1|_{x_0=1},\dots,G_k|_{x_0=1}\}$ is a standard basis for $\mathfrak{a}$ with respect to $>$.

Problem 4: Write a Singular procedure that, having as input a polynomial $f \in K[x]$, K field, and an integer $n \in \mathbb{N}$, returns the power series expansion of the inverse of f up to terms of degree n if f is a unit in $K[x]_>$ and 0 if f is not a unit.