TECHNISCHE UNIVERSITÄT
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Fachbereich Mathematik

Computer Algebra

Winter Semester 2013 - Problem Set 12

Due February 4, 2014, 12:00

Problem 1: Prove that any regular local ring is a domain.

HINT: Let A denote the ring and $\mathfrak{m}$ its maximal ideal. Make an induction on the dimension of A and try to find a non-zerodivisor in $\mathfrak{m} \setminus \mathfrak{m}^2$ which you can mod out.

Problem 2: Show that the definition of $\mathrm{Tor}_i^A(M, N)$ is independent of the free resolution of N.

Problem 3: Show that $\mathrm{Tor}_i^A(M, N) \cong \mathrm{Tor}_i^A(N, M)$ for two A-modules M, N.

Problem 4: Let $I, J \trianglelefteq A$ be two ideals. Prove that $\mathrm{Tor}_1^A(A/I, A/J) = (I \cap J)/(I \cdot J)$.