TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

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Computer Algebra

Winter Semester 2013 - Problem Set 11

Due January 28, 2014, 12:00

Problem 1: Let $(A, \mathfrak{m})$ be a regular local ring, $P \trianglelefteq A$ an prime ideal such that A/P is also a regular local ring. Prove that P can be generated by a regular sequence $x_1, \ldots, x_r$ in $\mathfrak{m} \setminus \mathfrak{m}^2$,

NOTE: A regular sequence is a sequence of elements $x_1, \ldots, x_k$ such that for all $i = 1, \ldots k$ the residue $\overline{x}_i \in R/\langle x_1, \ldots, x_{i-1}\rangle$ is a non-zerodivisor.

Problem 2:

(a) Let $f \in \langle x \rangle \trianglelefteq K[x]$ be irreducible, $x = (x_1, \ldots, x_n)$. Prove that

$$K[x]_{\langle x \rangle}/\langle f \rangle \text{ regular local ring} \quad \Longleftrightarrow \quad \langle \frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n} \rangle \not\subseteq \langle x \rangle$$

(b) Let $f \in K[x]$ be irreducible. Prove, without using that localization at prime ideals are regular, that $K[x]_{\langle f \rangle}$ is regular.

Problem 3: Write a SINGULAR procedure to compute the embedding dimension.

Problem 4: Assure yourself that it is possible to compute the gcd of two polynomials $f, g \in K[x_1, \ldots, x_n]$ using the syzygies of f and g. Create an example in SINGULAR which this method is faster than using the SINGULAR command `gcd`.