HOMEWORK 8

Define your terminology and explain notation. If you require a standard result, such as one of the Sylow theorems, then state it (or cite a result from your textbook) before you use it; otherwise give clear and complete proofs of your claims. The problems are of equal value, 6 points each. Partial solutions will be considered on their merits.

Exercise 1. Let E be a free module of rank n over a commutative ring R, and denote by $-^\vee = \operatorname{Hom}_R(-, R)$ the dualizing functor. Construct isomorphisms:

(a) $\bigwedge^k(E^\vee) \cong (\bigwedge^k E)^\vee$

(b) $\bigwedge^k E \cong (\bigwedge^{n-k} E)^\vee$

Exercise 2. Let R be a commutative ring.

(a) Show that any exact sequence $K \to M \to N \to 0$ of R-modules induces an exact sequence $\bigwedge K \otimes \bigwedge M \to \bigwedge M \to \bigwedge N \to 0$ of R-algebras.

(b) For R-modules M and N, show that $\bigwedge(M \oplus N) \cong \bigwedge M \otimes \bigwedge N$ as R-algebras.

Exercise 3.

(a) Compute $\operatorname{Tor}^i_{K[x,y]}(K, K)$ where K is a field.

(b) Compute $F_i\mathfrak{a}$ where $\mathfrak{a} = \langle x, y, z\rangle \leq K[x, y, z]$.

Exercise 4.

(a) Assume that $\mathbb{Q} \subseteq R$, and let M be an R-module. Define splittings of the canonical maps $\bigotimes^p M \to \bigwedge^p M$ and $\bigotimes^p M \to S^p M$.

(b) For a finite dimensional K-vector space V, show that $\bigotimes^2 V \cong S^2 V \oplus \bigwedge^2 V$.