**First Name:**____________________ **Last Name:**____________________

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**OKLAHOMA STATE UNIVERSITY**
**Department of Mathematics**

**MATH 2144 (Calculus I)**
Instructor: Dr. Mathias Schulze

**MIDTERM 1**
**February 9, 2011**

**Duration: 50 minutes**

**No aids allowed.**

This examination paper consists of **6** pages and **9** questions. Please bring any discrepancy to the attention of an invigilator. The number in brackets at the start of each question is the number of points the question is worth.

Answer **all** questions.

**To obtain credit, you must give arguments to support your answers.**

For graders' use:

| | Score |
|---|---|
| 1 (8) | |
| 2 (4) | |
| 3 (6) | |
| 4 (4) | |
| 5 (6) | |
| 6 (4) | |
| 7 (4) | |
| 8 (4) | |
| 9 (4) | |
| Total (44) | |

1. [8] Compute limits:

   (a) $\lim_{x\to -4} \frac{4^{-1}+x^{-1}}{4+x}$

   (b) $\lim_{t\to 0} \left(\frac{1}{t} - \frac{1}{t^2+t}\right)$

   (c) $\lim_{x\to -2} \frac{x+2}{x^3+8}$

   (d) $\lim_{h\to 0} \frac{\sqrt{1+h}-1}{h}$

**Solution:**

(a) $\lim_{x\to -4} \frac{4^{-1}+x^{-1}}{4+x} = \lim_{x\to -4} \frac{1}{4x} = -\frac{1}{16}$

(b) $\lim_{t\to 0} \left(\frac{1}{t} - \frac{1}{t^2+t}\right) = \lim_{t\to 0} \left(\frac{t+1-1}{t^2+t}\right) = \lim_{t\to 0} \frac{1}{t+1} = 1$

(c) $\lim_{x\to -2} \frac{x+2}{x^3+8} = \lim_{x\to -2} \frac{1}{x^2-2x+4} = \frac{1}{12}$

(d) $\lim_{h\to 0} \frac{\sqrt{1+h}-1}{h} = \lim_{h\to 0} \frac{1+h-1}{h(\sqrt{1+h}+1)} = \lim_{h\to 0} \frac{1}{\sqrt{1+h}+1} = \frac{1}{2}$

2. [4] Decide whether the function is continuous. Give arguments for your answer.

   (a) $f(x) = \ln|x-2|$

   (b) $f(x) = \begin{cases} \frac{x^2-x}{x^2-1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$

**Solution:**

(a) $f(x)$ is continuous: Its domain consists of two open intervals, $(-\infty, 2)$ and $(2, \infty)$. On $(-\infty, 2)$, $f(x) = \ln(2-x)$ which is continuous; on $(2, \infty)$, $f(x) = \ln(x-2)$ which is continuous, too.

(b) Since

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{x^2-x}{x^2-1} = \lim_{x\to 1} \frac{x}{x+1} = \frac{1}{2} \neq 1 = f(1),$$

$f(x)$ is not continuous at 1.

3. [6] Find all values for $a$ and $b$ such that the function

$$f(x) = \begin{cases} \frac{x^2-9}{x-3} & \text{if } x < 1 \\ (x-a)^2 & \text{if } 1 \leq x < 2 \\ 2ax - b & \text{if } 2 \leq x \end{cases}$$

becomes continuous.

**Solution:** First, note that $f(x) = x + 3$ for $x < 1$. Continuity is clear at $x \neq 1, 2$, as polynomials are continuous. The following two conditions are equivalent to continuity at 1 and 2 respectively:

$$4 = \lim_{x\to 1^-} f(x) = f(1) = (1-a)^2,$$
$$(2-a)^2 = \lim_{x\to 2^-} f(x) = f(2) = 4a - b.$$

The first equality gives $a = 1 \mp 2$, so $a = -1$ or $a = 3$. Then the second equality reads $5 \pm 4 = (1 \pm 2)^2 = 4 \mp 8 - b$ which gives $b = -1 \mp 12$. So either $a = -1$ and $b = -13$ or $a = 3$ and $b = 11$.

4. [4] Compute limits:

   (a) $\lim_{x\to-\infty}(x+\sqrt{x^2+2x})$

   (b) $\lim_{x\to\frac{\pi}{2}^+} e^{\tan x}$

**Solution:**

(a)

$$\begin{aligned}\lim_{x\to-\infty}(x+\sqrt{x^2+2x}) &= \lim_{x\to\infty}(\sqrt{x^2-2x}-x)\\ &= \lim_{x\to\infty}\frac{-2x}{\sqrt{x^2-2x}+x}\\ &= \lim_{x\to\infty}\frac{-2}{\sqrt{1-\frac{2}{x}}+1}\\ &= -1.\end{aligned}$$

(b)

$$\begin{aligned}\lim_{x\to\frac{\pi}{2}^+} e^{\tan x} &= \lim_{y\to-\infty} e^y\\ &= 0.\end{aligned}$$

5. [6] Find the horizontal and vertical asymptotes of the curves:

   (a) $y=\frac{x^2+1}{2x^2-3x-2}$.

   (b) $y=\frac{\sqrt{9x^6-x}}{x^3+1}$.

**Solution:**

(a) Horizontal asymptote: $y=\frac{1}{2}$
Vertical asymptotes: $x=2$, $x=-\frac{1}{2}$

(b) Horizontal asymptotes: $y=\pm 3$
Vertical asymptote: $x=-1$

6. [4] If $g(x) = 1 - x^3$, compute $g'(1)$ and use it to find the equation of the tangent line to the curve $y = 1 - x^3$ at the point $(1, 0)$.

**Solution:**

$$\begin{aligned} g'(1) &= \lim_{h \to 0} \frac{1 - (1+h)^3}{h} \\ &= \lim_{h \to 0} \frac{-3h - 3h^2 - h^3}{h} \\ &= \lim_{h \to 0} -3 - 3h - h^2 \\ &= -3. \end{aligned}$$

Tangent line: $y = -3(x - 1) = -3x + 3$.

7. [4] Find a function $f(x)$ and a number $a$ such that the following limit represents $f'(a)$:

(a) $\lim_{h \to 0} \frac{\sqrt[4]{16+h}-2}{h}$

(b) $\lim_{h \to 0} \frac{\cos(\pi+h)+1}{h}$

**Solution:**

(a) $f(x) = \sqrt[4]{x}$, $a = 16$.

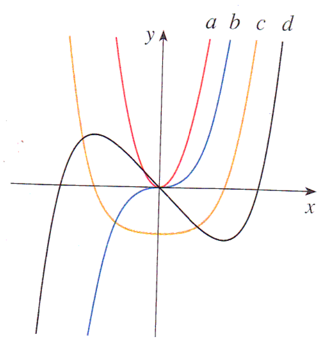
(b) $f(x) = \cos(x)$, $a = \pi$.

8. [4] Compute $f'(x)$ using the limit definition for $f(x) = x^3$.

**Solution:**

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} \\ &= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \\ &= \lim_{h \to 0} 3x^2 + 3xh + h^2 \\ &= 3x^2. \end{aligned}$$

9. [4] The figure shows the graphs of $f(x)$, $f'(x)$, $f''(x)$, $f'''(x)$. Identify each curve.




**Solution:**

(a) is the graph of $f'''(x)$.
(b) is the graph of $f''(x)$.
(c) is the graph of $f'(x)$.
(d) is the graph of $f(x)$.

**End of examination**
**Total pages: 6**
**Total marks: 44**