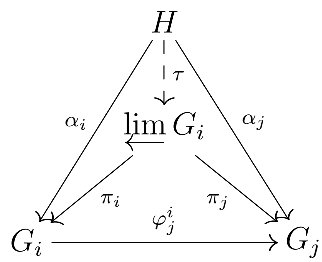
HOMEWORK 6

Define your terminology and explain notation. If you require a standard result, such as one of the Sylow theorems, then state it (or cite a result from your textbook) before you use it; otherwise give clear and complete proofs of your claims. The problems are of equal value, 6 points each. Partial solutions will be considered on their merits.

Exercise 1. Let $\{G_i\}$ be an inverse system with maps $\{\varphi^i_j\}_{i\geq j}$. Prove that $\varprojlim G_i$ with its natural projections π_i has the following universal property: For all H and $\{\alpha_i\}_{i\in I}$ making the large triangle



commutative for all $i \geq j$, there is a map τ making the left and right small triangles commutative.

Exercise 2.

(a) Describe the group $\varprojlim(\cdots \hookrightarrow G_{i+1} \hookrightarrow G_i \hookrightarrow G_{i-1} \hookrightarrow \cdots)$ for given subgroups $G_{i+1} \leq G_i \leq G$ of a fixed group G.

(b) Let $\mathscr{H}$ be a family of subgroups of G as in the definition of a profinite group. Let $\{H_i\}_{i\in\mathbb{N}} \subset \mathscr{H}$ be a cofinal subset, that is, for all $H \in \mathscr{H}$ there is an $i \in \mathbb{N}$ such that $H_i \subseteq H$. Show that $\varprojlim_{i\in\mathbb{N}}(G/H_i) \cong \varprojlim_{H\in\mathscr{H}} G/H$.

Exercise 3. Show that $\varprojlim$ is a covariant functor. Outline:

(a) Describe what is a map of inverse systems.

(b) Define what is $\varprojlim$ applied to such a map.

(c) Show that $\varprojlim$ commutes with composition of maps.

Exercise 4. Assume that $1 \to \{A_i\} \to \{B_i\} \to \{C_i\} \to 1$ is an exact sequence of inverse systems.

(a) Show that $1 \to \varprojlim\{A_i\} \to \varprojlim\{B_i\} \to \varprojlim\{C_i\}$ is exact. (So $\varprojlim$ is a left exact functor.)

(b) Assume that $\{A_i\} = (\{A_i\}, \{f_{i,j}\})$ satisfies the *Mittag–Leffler condition*: For all k there is a $j \geq k$ such that $f_{k,j}(A_j) = f_{k,i}(A_i)$ for all $i \geq j$. Show that $1 \to \varprojlim\{A_i\} \to \varprojlim\{B_i\} \to \varprojlim\{C_i\} \to 1$ is exact.